ACOUSTICAL LETTER

Impulse response measurement that maximizes signal-to-noise ratio against ambient noise

Naoya Moriya* and Yutaka Kaneda†

Department of Information and Communication Engineering, School of Engineering, Tokyo Denki University, 2–2 Kanda-Nishiki-cho, Chiyoda-ku, Tokyo, 101–8457 Japan

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1. Introduction

Impulse response measurement is very important in acoustic measurement and signal processing. However, measurement errors occur due to ambient noise. M-sequence [1] and swept-sine signals (such as TSP signal [2]) are widely used to solve this problem. In recent years, effective signals have been proposed for impulse response measurement in the presence of low-frequency ambient noise [3,4]. In this study, we produced measurement signals that have maximal signal-to-noise (SN) ratios against ambient noise with an arbitrary spectrum.

2. Optimal spectrum of measurement signal

2.1. Noise-induced error

Figure 1(a) shows the principle of impulse response measurement in the frequency domain. A signal with the frequency spectrum $S(\omega)$ (hereafter abbreviated as S) is applied to a target system whose transfer function is H. By multiplying the observed output $H \cdot S$ by $1/S$, the transfer function H of the system is obtained (the inverse Fourier transform of H represents its impulse response).

Figure 1(b) shows the case in which the ambient noise N_0 is added to the observed output $H \cdot S$. Here, the noise is assumed to be stationary. In this case, N_0/S is added to the measurement result as a noise-induced error.

2.2. Optimal spectrum

Let $\omega(= 1, 2, \ldots, N)$ be the discrete frequency. The energy spectrum of the measurement signal S is denoted as $P_S(\omega)(= |S(\omega)|^2)$. The entire energy of the signal is given by

$$P_{St} = \sum_{\omega=1}^{N} P_S(\omega). \tag{1}$$

Furthermore, by denoting the energy spectrum of the ambient noise as $P_N(\omega)(= E[|N_0(\omega)|^2]$, where $E[\,\cdot\,]$ is the expectation value), the entire energy P_{Nt} of the noise-induced error N_0/S is

$$P_{Nt} = \sum_{\omega=1}^{N} P_N(\omega)/P_S(\omega). \tag{2}$$

Here, assuming that the entire energy of the measurement signal P_{st} is kept constant at C, the energy spectrum of the measurement signal, $P_{Sopt}(\omega)$, that makes noise-induced error (P_{Nt}) minimum is derived by solving simultaneous equations consisting of unknown $P_s(\omega)$ (where $\omega = 1, 2, \ldots, N$) and λ [5]

$$\frac{\partial}{\partial P_s(\omega)}(P_{Nt} + \lambda(P_{St} - C)) = 0. \tag{3}$$

Then, the optimum spectrum $P_{Sopt}(\omega)$ is given as

$$P_{Sopt}(\omega) = \frac{C\sqrt{P_N(\omega)}}{\sqrt{P_N(1)} + \sqrt{P_N(2)} + \cdots + \sqrt{P_N(M)}}. \tag{4}$$

From this equation, it is clear that the energy spectrum of the signal that minimizes the noise-induced error and maximizes the SN ratio, $P_{sopt}(\omega)$, is proportional to the square root of the energy spectrum of noise, $P_N(\omega)$.

2.3. Synthesis of optimum measurement signal

The two commonly used representative signals are M-sequence and swept-sine signals. Generally, the spectrum of these signals is white. However, the noise-induced error can be minimized by introducing the optimum spectrum, as discussed in section 2.2.

• Random signal

Prior to actual calculations, the energy spectrum of the noise, $P_N(\omega)$, is measured and a random signal that has the fourth-order root of the spectrum $P_N(\omega)$ as its amplitude is synthesized. More practically, one period of an M-sequence signal is converted into a signal in the frequency domain. We then multiply the frequency-domain signal by the optimal-amplitude spectrum (the fourth-order root of $P_N(\omega)$). Thereafter, the signal is reconverted back to a time-domain signal and we designate it as the measurement signal.

• Swept-sine signal

If the amplitude characteristics are altered while maintaining the phase characteristics of a swept-sine signal, the constant-amplitude characteristics of its waveform, which are the advantage of this approach, disappear. Therefore, we produce the optimum energy spectrum $P_{Sopt}(\omega)$ by adjusting the sweeping time for each frequency. By utilizing the fact that the phase characteristics of a constant-amplitude swept-sine signal can be obtained if the energy spectrum is integrated twice, we can obtain the desired swept-sine signal [5].

*Current affiliation: Yamaha Co.
†e-mail: kaneda@c.dendai.ac.jp

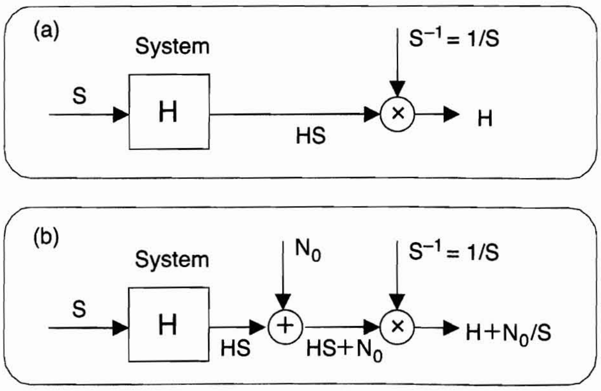

Fig. 1 Principle of impulse response measurement and noise-induced error characteristics: (a) without ambient noise, (b) with ambient noise.

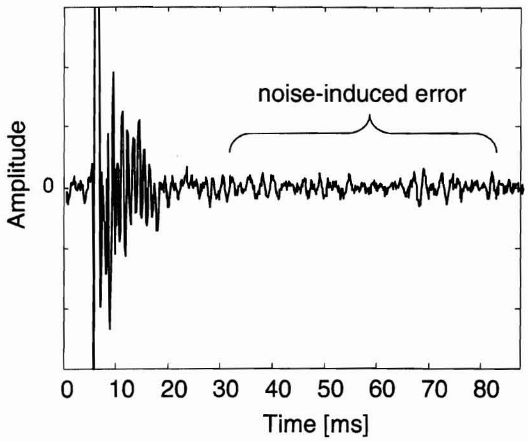

Fig. 2 Measurement result contaminated by noise-induced error.

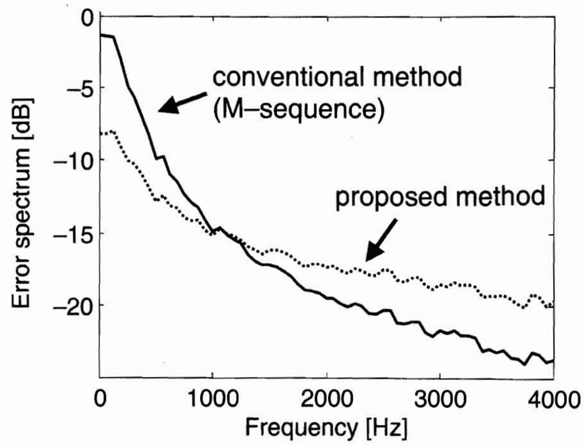

Fig. 3 Comparison of noise-induced error spectra when ambient noise is low-frequency noise.

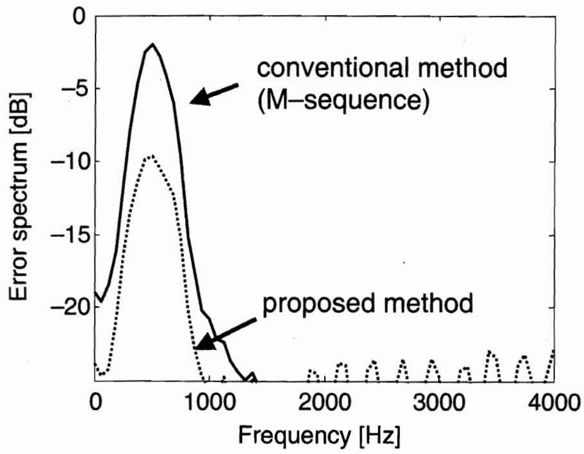

Fig. 4 Comparison of noise-induced error spectra when ambient noise is band noise.

3. Error reduction simulation

3.1. Simulation conditions

To confirm the effectiveness of the proposed method, we simulated the procedure. The object of measurement in this study is an impulse response of a loudspeaker. In the conventional method, an M-sequence signal with length 2^{13}-1 was used. In the proposed method, a signal in which the optimal spectrum is assigned to the M-sequence was used. Furthermore, a) low-frequency noise and b) 500–1,000 Hz band noise are added to the signal as ambient noise.

3.2. Results

Figure 2 shows an example of the measurement results. Measurement errors like a stationary noise contaminate the impulse response. We calculated the spectrum of the noise-induced errors included in each measurement result.

Figure 3 shows the results for the case of a low-frequency noise. When the conventional method is used, the error spectrum is the same as that of ambient noise $N_0(\omega)$ because $S \equiv 1$ in Fig. 1(b). On the other hand, in the proposed method, the low-frequency component, the energy of which is large, was suppressed. The noise suppression effect of the proposed method for the entire band was approximately 3 dB.

Figure 4 shows the results for the case of band noise. In the proposed method, the error spectrum is reduced in a manner similar to that in the case shown in Fig. 3, although it is slightly enlarged in the high-frequency range. The noise suppression effect for the entire band was approximately 6 dB.

4. Conclusions

We proposed a spectrum of an impulse response measurement signal that maximizes the SN ratio of the measurement result to ambient noise. In practice, this method is used by synthesizing random and swept-sine signals with the above spectrum. In our simulations, we were able to reduce noise-induced errors by approximately 3–6 dB.

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